

# Summation 5

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Q) Let  $A = \underbrace{666 \dots 666}_{2016 \text{ times}}$  and  $B = \underbrace{999 \dots 999}_{2016 \text{ times}}$ . Let  $N = A \times B$

Find the sum of digits of  $N$ .

Ans: -  $A = \sum_{i=0}^{2015} 6 \times 10^i$        $B = \sum_{j=0}^{2015} 9 \times 10^j$

$$\left( \sum 6 \times 10^i \right) \left( \sum 9 \times 10^j \right)$$

$i+j = 2016$   
 $i = 2015, j = 1$   
 $i = 2014, j = 2 \dots$

$$= \sum_{\substack{i, j \\ \in \{0, \dots, 2015\}}} (54 \times 10^{i+j})$$

↳ number of terms is  $2015^2$

$$A = \sum_i 6 \times 10^i$$

$$B = 10^{2016} - 1$$

$$AB = \left( \sum_i 6 \times 10^i \right) (10^{2016} - 1) = 10^{2016} \sum_i 6 \times 10^i - \sum_i 6 \times 10^i$$

$$= \sum_i 6 \times 10^{2016+i} - \sum_i 6 \times 10^i$$

$$= \underbrace{66 \dots 60 \dots 0}_{2016} - \underbrace{6 \dots 6}_{2016}$$

$$= \underbrace{66 \dots 6}_{2015} \underbrace{533 \dots 34}_{2015}$$

So sum of digits

$$= 6 \times 2015 + 5 + 3 \times 2015 + 4$$